Technical Comments

AIAA 82-4131

Comment on "Optimal Feedback Slewing of Flexible Spacecraft"

J.L. Junkins
Virginia Polytechnic Institute and State University, Blacksburg, Va.

IN Ref. 1, Breakwell formulates optimal controls which minimize

$$J=\frac{1}{2}\int_0^{t_f}\left(X^TAX+U^TBU\right)\mathrm{d}t \tag{1}$$

subject to the linear autonomous state differential equation

$$\dot{X}=FX+GU \tag{2}$$

with

$$X(0) \text{ and } X(t_f) \text{ prescribed} \tag{3}$$

The resulting optimal control is determined therein from

$$U=-B^{-1}G^T\lambda \tag{4}$$

with

$$\begin{Bmatrix}\dot{X}\\ \dot{\lambda}\end{Bmatrix}=\begin{bmatrix}F & -GB^{-1}G^T\\ -A & -F^T\end{bmatrix}\begin{Bmatrix}X\\ \lambda\end{Bmatrix} \tag{5}$$

Breakwell considers the slewing of a flexible spacecraft whose dynamics can be approximated by Eq. (2). In his conclusions, he discusses two problems encountered previously by the author in Refs. 2 and 3. These are:

1) The fact that the differential equation(s) are stiff and often subject to numerical difficulties if one employs eigenvalue routines or attempts to numerically integrate the associated state transition matrix.

2) The fact that significant vibrational energy is imparted to the higher modes, even though n of them can be arrested upon arrival at time t_f (which is less comforting in the presence of modeling and measurement errors).

Regarding the first problem, Ref. 4 documents an attractive way to obtain high-precision state transition matrices using diagonal Pade approximations in conjunction with the matrix exponential identity

$$e^{Ct}=(e^{Ct/2^n})^{2^n} \tag{6}$$

In Ref. 3, it was applied to essentially the same problem addressed in the present discussion. (For maneuvering of a flexible spacecraft, up to ten modes were considered; the state transition matrix was typically calculated with ten-digit precision.)

Regarding the second problem, the fundamental source of the high-frequency excitation of the vibratory degrees of freedom is the jump discontinuity (initially and finally) in the control torques determined from Eq. (4) (see Figs. 2-7 of Ref. 1). This jump discontinuity can be eliminated easily with a modest modification of the formulation. If, instead of minimizing Eq. (1), we choose the index

$$J=\frac{1}{2}\int_0^{t_f}\left[X^TAX+U^TB_0U+\dot{U}^TB_1\dot{U}+\ddot{U}^TB_2\ddot{U}\right]\mathrm{d}t \tag{7}$$

where the overdot indicates a time derivative. Then we still have Eq. (5) but, in lieu of the *algebraic* equation (4), the optimal control satisfied the *differential* equation

$$\frac{\mathrm{d}^4U}{\mathrm{d}t^4}=B_2^{-1}\left\{-G^T\lambda-B_0U+B_1\frac{\mathrm{d}^2U}{\mathrm{d}t^2}\right\} \tag{8}$$

and we are free to impose the four *control boundary conditions*

$$U(0)=0 \quad U(t_f)=0 \quad \frac{\mathrm{d}U}{\mathrm{d}t}(0)=0 \quad \frac{\mathrm{d}U}{\mathrm{d}t}(t_f)=0 \tag{9}$$

Equations (9) require that the controls be turned on and off smoothly. The maneuvers resulting from Eq. (8) typically require a bit more energy than those using Eq. (4), but the amplitude of the vibrations will typically be reduced by an order of magnitude.

Numerical experience suggests choosing the weight B_2 large enough to dominate the calculations; all of the B_i should be positive-definite-symmetric.

References

[1]Breakwell, J.A. "Optimal Feedback Slewing of Flexible Spacecraft", *Journal of Guidance and Control*, Vol. 4, Sept-Oct. 1981, pp. 472-479.

[2]Turner, J.D. and Junkins, J.L., "Optimal Large Angle Rotational Maneuvers of Flexible Spacecraft", *Journal of Guidance and Control*, Vol. 3, Nov.-Dec. 1980, pp. 578-585.

[3]Turner, J.D. and Chun, H.M., "Optimal Distributed Control of a Flexible Spacecraft During A Large-Angle Rotational Maneuver," Proceedings of the Third VPI&SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft, Blacksburg, Va., June 1981.

[4]Ward, R.C., "Numerical Computation of the Matrix Exponential with Accuracy Estimate", *Journal of Numerical Analysis*, Vol. 14, Sept. 1977, pp. 600-610.

Received Oct. 9, 1981.

*Professor, Department of Engineering Science and Mechanics. Associate Fellow AIAA.

AIAA 82-4132

Comment on "Optimal Control via Mathematical Programming"

Lincoln J. Wood*
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, Calif.

IN Ref. 1, Sheela and Ramamoorthy discuss the numerical solution of a minimum-time low-thrust orbit transfer problem, using a form of the Ritz method.[2,3] The optimized transfer times which they obtain vary over a wide range as the orders of certain series expansions and the time intervals over

Received July 27, 1981. Copyright © American Institute of Aeronautics and Astronautics, Inc., 1981. All rights reserved.

*Supervisor, Future Mission Studies Group, Navigation Systems Section. Member AIAA.